Varianta 071

Subiectul I:

a) $d=\sqrt{2}$. b) $\left|\frac{1+i}{1-i}\right|=1$. c) $\cos\frac{\pi}{2}\cdot\sin\frac{\pi}{2}=0$ .d) $\sin x=\frac{\sqrt{3}}{2}$ e) x-y+3z-3=0.f) $x+y=2$.

Subiectul II:

1. a) 6 functii b) *f(g(x))*= x^2 .c) C_5^3=10. d) $\hat{1}+\hat{3}+\hat{5}+...+\widehat{11}=\hat{0}$

e) $\hat{x}\in\mathbf{Z}_{12}:\widehat{x^2}=\hat{x}\Rightarrow\hat{x}\in\{\hat{0},\hat{1},\hat{4},\hat{9}\}\Rightarrow P(e)=\frac{4}{12}=\frac{1}{3}$

2. a) $\lim_{x\to\infty}f(x)=\lim_{x\to\infty}e^x(1-\frac{x}{e^x}-\frac{1}{e^x})=\infty$. b) f'(x)=e^x-1.

c) f''(x)=$e^x>0,(\forall)x\in R$ => f convexă pe **R**.

d) din (b)=> f'(x)$\geq 0,(\forall)x\geq 0$ => f strict crescatoare pe $[0,\infty]$

e) din f'<0, $(\forall)x<0$ si d) => f'(0)=0 si isi schimba semnul in vecinatatea lui 0 =>

x_0 punct de minim local pentru. f

=> $f(x)\geq 0,(\forall)x\in\mathbf{R}\Leftrightarrow e^x-x-1\geq 0,\forall x\in\mathbf{R}\Leftrightarrow e^x\geq x+1;(\forall)x\in\mathbf{R}$

Subiectul III:

a)Verificare

b) Fie A,B$\in G$

Avem $(A\cdot B)\cdot(AB)^t=(AB)\cdot(B^tA^t)=A(BB^t)A^t=AA^t=I_3 si \det(A\cdot B)=\det A\det B>0$ =>

$(AB)\in G,(\forall)A,B\in G$

c) $A\cdot A^t=A^t\cdot A=I_3\Rightarrow A^{-1}=A^t$

d) $\det(A\cdot A^t)=\det A\cdot\det A^t=(\det A)^2$ dar $\det(A\cdot A^t)=\det I_3=1$=>

$(\det A)^2=1,\det A>0\Rightarrow\det A=1$

e) $\forall A\in G$ avem $(A^t-I_3)\cdot A=A^t\cdot A-I_3\cdot A=I_3-A$

f) Din e) $\det(A^t-!_3)\det A=\det(I_3-A)$ si $\det(A^t-!_3)=\det(A-I_3)=-\det(I_3-A)$ =>

=> $\det(I_3-A)=-\det(I_3-A)\Rightarrow\det(A-I_3)=0$.

g) Fie A=$\begin{pmatrix}a&b&c\\m&n&p\\n&v&t\end{pmatrix}\in G$ si $X=\begin{pmatrix}x\\y\\z\end{pmatrix}\neq O_3, a.i. A\cdot X=X\Leftrightarrow A\cdot X-X=O_3\Rightarrow$

sistemul are matricea $(A-I_3)\Rightarrow\det(A-I_3)=0$. Deoarece sistemul este liniar şi omogen, admite şi solutii diferite de cea banală $\Rightarrow(\forall)A\in G,(\exists)X\in M_3(\mathbf{R})a.i.AX=X$

şi X$\neq O_{3,1}$

Subiectul IV:

a) $f_1(x) = [(x^2 - 1)]' = 2x, (\forall) x \in \mathbf{R}$.

b) $f_2'(x) = 24x$.

c) $\int_{-1}^{1} f_1(x)dx = 0$.

d) $f_n(x) = \left[\left(x^2 - 1\right)^n\right]^{(n)} = (x^{2n} + ...)^{(n)} = (2n)(2n-1)\cdot ... \cdot (n+1)x^n + ... \Rightarrow a_n = \frac{(2n)!}{n!}$

e) $f_n(x) = \left[(x-1)^n (x+1)^n\right]^{(n)} = \left[t_n(x)\right]^{(n)}$

Functia $t_n(x)$ are pe 1 si pe -1 radacini multiple de ordinul n, de unde rezulta relatia ceruta.

f) Integram succesiv prin parti :

$$\int_{-1}^{1} f_n(x)\cdot g(x)dx = \int_{-1}^{1} \left[\left(x^2 - 1\right)^n\right]^{(n)} \cdot g(x)dx = \int_{-1}^{1} \left[\left(t_n^{(n-1)}(x)\right)\right]' \cdot g(x)dx =$$

$$= t_n^{(n-1)}(x)\cdot g(x)\Big|_{-1}^{1} - \int_{-1}^{1} t_n^{(n-1)}(x)\cdot g'(x)dx = 0 - \int_{-1}^{1} \left[\left(t_n^{(n-2)}(x)\right)\right]' \cdot g'(x)dx =$$

$$= -t_n^{(n-2)}(x)\cdot g'(x)\Big|_{-1}^{1} + \int_{-1}^{1} t_n^{(n-2)}(x)\cdot g''(x)dx = 0 + \int_{-1}^{1} t_n^{(n-2)}(x)\cdot g''(x)dx = ... = (-1)^n \int_{-1}^{1} t_n(x)\cdot g^{(n)}(x)dx$$

g) Daca luam $g = h$ in f) avem $g^{(n)}(x) = 0, x \in [-1,1]$, deci $\int_{-1}^{1} f_n(x)\cdot h(x)dx. = 0$.